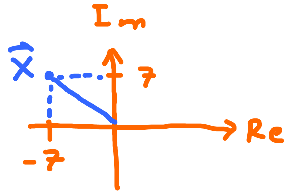
Q1 [F2010] Phasor to Sinusoid

Tuesday, August 20, 2013 8:25 PM

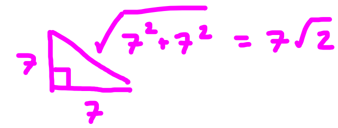
(a) $\vec{x} = -7 + 7j$



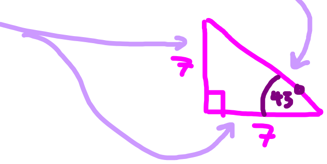
$= 7\sqrt{2} \angle 135^\circ$

Note that you should be able to get this without using the calculator.

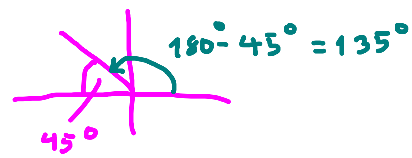
$7\sqrt{2}$ is from Pythagoras' theorem:



Also, we know that the angle in the triangle is 45° because the two sides have the same length.



Therefore, the phase is



$\Leftrightarrow x(t) = 7\sqrt{2}\cos(100t + 135^\circ)$

ω is given.

(b)

$$x(t) = \underbrace{7\cos(t - 777^\circ)}_{\substack{= 7\cos(t - 777^\circ + 720^\circ) \\ = 7\cos(t - 57^\circ)}} - \overbrace{7\sin(t - 77^\circ)}^{\substack{= 7\cos(t - 77^\circ - 90^\circ + 180^\circ) \\ = 7\cos(t + 13^\circ)}}$$

Phasor form: $\vec{x} = 7\angle 57^\circ + 7\angle 13^\circ \approx 3.812 - 5.37j + 6.821 + 1.56j$

$= 10.633 - 4.3j \approx 11.47 \angle -22^\circ$

$= 10.633 - 4.3j \approx 11.47 \angle -22^\circ$

Conversion back to time domain:

$x(t) = 11.47 \cos(t - 22^\circ)$

Q2 [Alexander and Sadiku, 2009, Q9.24a]

Tuesday, August 27, 2013 7:32 PM

Recall that $v(t) \Leftrightarrow \vec{V}$

$$\frac{d}{dt} v(t) \Leftrightarrow j\omega \vec{V}$$

$$\int v(t)\,dt \Leftrightarrow \frac{\vec{V}}{j\omega}$$

For this question,

$$v + \int v\,dt = 5\cos(\underset{\omega = 1}{t} + 45^\circ)$$

Step 1: Conversion to phasor rep.

$$\Updownarrow$$

$$\vec{V} + \frac{\vec{V}}{j\omega} = 5 \angle 45^\circ$$

Step 2: Solve for the variable under consideration

$$\vec{V}\left(1 + \frac{1}{j}\right) = 5 \angle 45^\circ$$

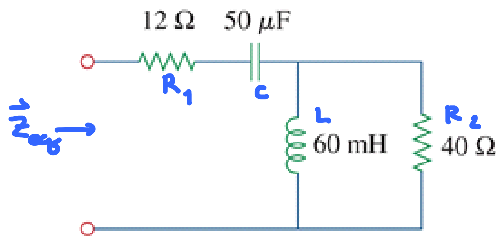
$$\vec{V} = \frac{5 \angle 45^\circ}{1-j} = \frac{5 \angle 45^\circ}{\sqrt{2} \angle -45^\circ} = \frac{5}{\sqrt{2}} \angle 90^\circ$$

Step 3: Conversion back to time domain

$$v(t) = \frac{5}{\sqrt{2}} \cos(t + 90^\circ) \approx 3.536 \cos(t + 90^\circ)$$

Q3 [Alexander and Sadiku, 2009, Q9.56]

Sunday, August 25, 2013 9:59 PM



$\omega = 377$

$\vec{Z}_C = \frac{1}{j\omega C} \approx -53.05j$

$\vec{Z}_L = j\omega L \approx 22.62j$

$\vec{Z}_{eq} = (R_1 + \vec{Z}_C) + \frac{\vec{Z}_L R_2}{\vec{Z}_L + R_2}$

$\approx 21.69 - 35.91j \ \Omega$

Note that, by convention, the impedance is answered in rectangular form instead of polar form.

(In contrast, phasor is answered in polar form.)

Q4 [F2010] AC Analysis

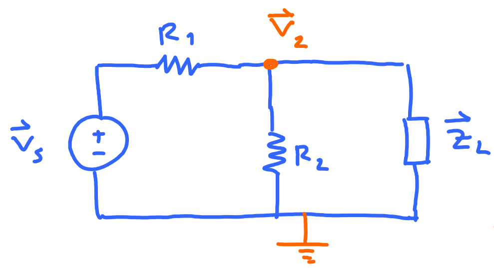
Monday, August 26, 2013 11:05 AM

(a) $\omega = 200$ [rad/s]

$v_s(t) = 7\cos(200t + 30^\circ)$ V $\Leftrightarrow$ $V_s = 7 \angle 30^\circ$ V

(b) $\vec{Z}_L = j\omega L = j \times 200 \times 5\text{ m} = j\ \Omega$

(c)



Nodal Analysis:

$$\frac{\vec{V}_2 - \vec{V}_s}{R_1} + \frac{\vec{V}_2}{R_2} + \frac{\vec{V}_2}{\vec{Z}_L} = 0$$

$$\frac{\vec{V}_2 - 7\angle 30^\circ}{6} + \frac{\vec{V}_2}{4} + \frac{\vec{V}_2}{j} = 0$$

$$\Rightarrow \vec{V}_2 = -0.138 + 1.068j = 1.077 \angle 97.38^\circ$$

$$v_2(t) = 1.077\cos(200t + 97.38^\circ)\text{ V.}$$

ω from above

(d) Method 1: Use the answer from the previous part

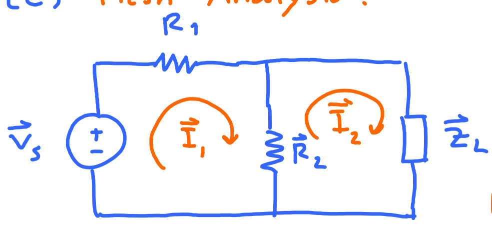
$$\vec{V}_1 = \vec{V}_s - \vec{V}_2 \approx 6.201 + 2.43j \approx 6.66 \angle 21.4^\circ \text{ V}$$

Method 2: Voltage divider: $\frac{R_1}{R_1 + R_2 /\!/ \vec{Z}_L}\vec{V}_s \approx 6.66 \angle 21.4^\circ$ V

$$v_1(t) = 6.66\cos(200t + 21.4^\circ)\text{ V}$$

ω from above

(e) Mesh Analysis.



Mesh #1: $\vec{V}_s - \vec{I}_1 R_1 - (\vec{I}_1 - \vec{I}_2)R_2 = 0$

$$7\angle 30^\circ - 6\vec{I}_1 - 4(\vec{I}_1 - \vec{I}_2) = 0$$

Mesh #2: $-(\vec{I}_2 - \vec{I}_1)R_2 - \vec{I}_2 \vec{Z}_L = 0$

$$-4(\vec{I}_2 - \vec{I}_1) - j\vec{I}_2 = 0 \Rightarrow 4\vec{I}_1 - (4+j)\vec{I}_2 = 0$$

$$\vec{I}_1 = \left(1 + \frac{j}{4}\right)\vec{I}_2.$$

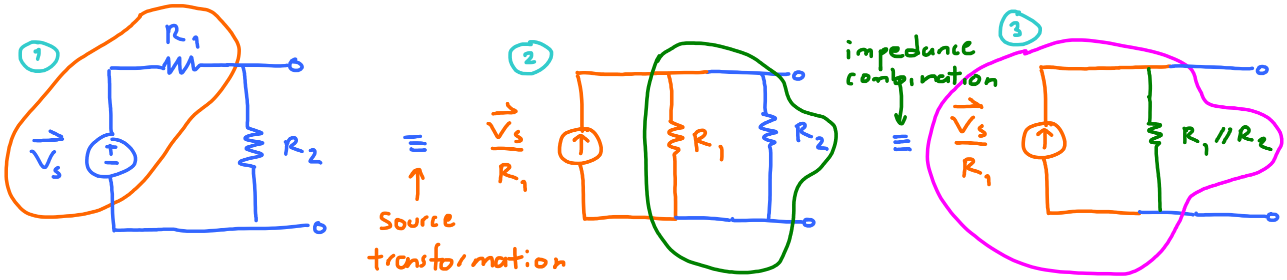
$$\vec{I}_1 = 1.03 + 0.4j = 1.11 \angle 21.4^\circ \text{ A}$$

$$\vec{I}_2 = 1.07 + 0.138j = 1.077 \angle 7.38^\circ \text{ A}$$

(f) $\vec{I}_L = \vec{I}_2 = 1.077 \angle 7.38^\circ \text{ A} \iff i_L(t) = 1.077 \cos(200t + 7.38^\circ) \text{ A}$

ω from above

(g)



4

$\equiv \frac{\vec{V}_s}{R_1} \times (R_1 // R_2)$

source transformation

$R_1 // R_2 = 2.4\,\Omega \Rightarrow \vec{Z}_A = 2.4\,\Omega$

$= \vec{V}_s \frac{R_2}{R_1 + R_2}$

$= 2.425 + 1.4j = 2.8 \angle 30^\circ \text{ V} \Rightarrow \vec{V}_A = 2.8 \angle 30^\circ \text{ V}$

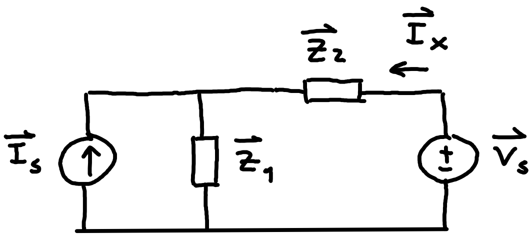
(h) $\vec{I}_L = \frac{\vec{V}_A}{\vec{Z}_A + \vec{Z}_L} = \frac{2.8 \angle 30^\circ}{2.4 + j} = \frac{2.8 \angle 30^\circ}{2.6 \angle 22.62^\circ} = 1.077 \angle 7.38^\circ$

$= 1.068 + 0.138j$

$\iff i_L(t) = 1.077 \cos(200t + 7.38^\circ) \text{ A}$

Q5 [Alexander and Sadiku, 2009, Q10.43] Superposition

Sunday, August 25, 2013 9:59 PM



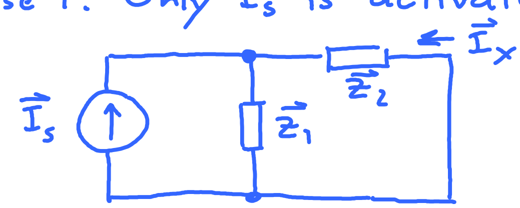
For this question, both i_s and v_s have the same (angular) frequency which is $\omega = 2$.

Hence, $\vec{Z}_1 = j\omega L = j \times 2 \times 4 = 8j$

$$\vec{Z}_2 = \frac{1}{j\omega C} + 3 = \frac{1}{j \times \cancel{2} \times \frac{1}{\cancel{8}_4}} + 3 = \frac{4}{j} + 3 = 3 - 4j$$

We also have $\vec{I}_s = 5 \angle 10°$ and $\vec{V}_s = 10 \angle -60°$.

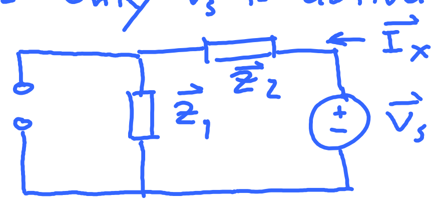
Case 1: Only $\vec{I}_s$ is activated



$$\vec{I}_x = -\frac{\vec{Z}_1}{\vec{Z}_1 + \vec{Z}_2}\vec{I}_s$$

current divider

Case 2: Only $\vec{V}_s$ is activated



$$\vec{I}_x = \frac{\vec{V}_s}{\vec{Z}_1 + \vec{Z}_2}$$

impedance combination & Ohm's law

Now, because the two cases have the same frequency, we can combine the phasors.

We can also convert them into sinusoids and then combine. The answer will be the same.

Caution If the frequencies are not the same, do not combine the phasors. You must convert them into sinusoids first.

$$\vec{I}_x = -\frac{\vec{Z}_1}{\vec{Z}_1 + \vec{Z}_2}\vec{I}_s + \frac{\vec{V}_s}{\vec{Z}_1 + \vec{Z}_2} = \frac{\vec{V}_s - \vec{Z}_1\vec{I}_s}{\vec{Z}_1 + \vec{Z}_2} = -6.26 - 7.68j = 9.9 \angle -129.17°$$

$$i_x(t) = 9.9\cos(2t - 129.17°)\ \text{A}$$

Q6 [Alexander and Sadiku, 2009, Q10.58] Thevenin

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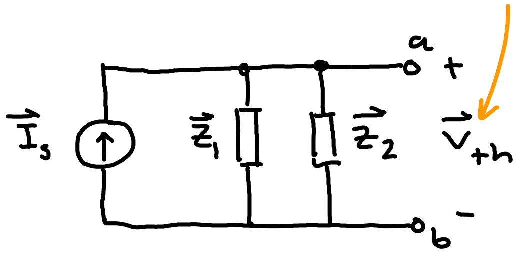
Recall that this is the open-circuit voltage



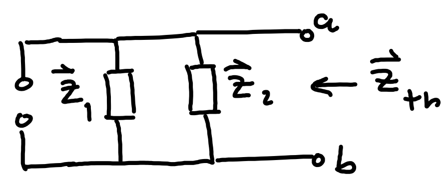
$\vec{I}_s = 2\angle 30^\circ = 1.732 + j$ A

$\vec{Z}_1 = 8 - 6j$ Ω The impedance of the resistor and capacitor are combined here.

$\vec{Z}_2 = 10j$ Ω

$$\vec{V}_{th} = \vec{I}_s \times (\vec{Z}_1 /\!/ \vec{Z}_2) = 12.32 + 18.7j = 22.36\angle 56.6^\circ \text{ V}$$

To find $\vec{Z}_{th}$, the source is deactivated, leaving only $\vec{Z}_1$ and $\vec{Z}_2$. The equivalent impedance at terminals a-b is



$$\vec{Z}_{th} = \vec{Z}_1 /\!/ \vec{Z}_2 = \frac{\vec{Z}_1 \vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} = 10 + 5j = 5(2+j)\ \Omega$$